Technical Comment

Comments on "Attenuation of Inlet Flow Distortion Upstream of Axial Flow Compressors"

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THE paper by Callahan and Stenning[1] reports an investigation of the three-dimensional upstream attenuation of inlet axial velocity distortion by an axial flow compressor. However, the conclusion presented concerning the influence of inlet Mach number on the distortion attenuation is not completely correct, as the upstream perturbation velocity field does in fact depend on the value of the mean flow Mach number. This discrepancy can be significant for compressors with high (>0.5, say) inlet flow Mach numbers.

In addition, it is shown below that in general for a compressor without inlet guide vanes, the distorted flow performance can be affected not only by the perturbations in inlet axial velocity, as assumed in Ref. 1, but by the asymmetric circumferential velocity components associated with the upstream flow shift which is induced by the compressor.

In order to examine the first of these points, let us study the simpler case of the asymmetric two-dimensional flow field upstream of a compressor of high hub-tip ratio. Using the notation of Ref. 1, consider a "far upstream" (i.e. at upstream infinity) axial velocity distribution of the form

$$U = \bar{U} + \sum_n \varepsilon_n \cos n\theta; \qquad \left|\frac{\varepsilon_n}{\bar{U}}\right| \ll 1 \tag{1}$$

(The barred quantities denote mean flow variables, which are constant in the upstream region).

There will be an upstream irrotational velocity field induced by the compressor in response to the above flow distortion. Adapting the analysis presented in Ref. 1 to the two-dimensional situation, the equation for the velocity potential, φ, associated with this irrotational field is

$$(1/r_m^2)\partial^2\varphi/\partial\theta^2 + (1-\bar{M}^2)\partial^2\varphi/\partial x^2 = 0 \tag{2}$$

In this equation r_m is some mean radius at which we "unroll" the compressor annulus to obtain the two-dimensional region. The solution of Eq. (2) satisfying boundary conditions analogous to those given in Ref. 1 is

$$\varphi = \sum_n A_n \exp[nx/r_m(1-\bar{M}^2)^{1/2}] \cos n\theta \tag{3}$$

where the A_n are constants which are dependent on the slope of the compressor characteristic. The irrotational perturbation velocity components can thus be expressed as

$$u = (1/r_m)\sum_n [nA_n/(1-\bar{M}^2)^{1/2}] \exp[nx/r_m(1-\bar{M}^2)^{1/2}] \cos n\theta \tag{4}$$

$$v = -(1/r_m)\sum_n nA_n \exp[nx/r_m(1-\bar{M}^2)^{1/2}] \sin n\theta \tag{5}$$

In the above expression for u there is a factor of $[1/(1-\bar{M}^2)^{1/2}]$ which is absent from the expression for u (Eq. 18) given in Ref. 1. Alternatively, if one takes the expression for u as a starting point, then the equations in Ref. 1 which define v, w, and φ (Eqs. 17, 19 and 20) should include a factor of $(1-\bar{M}^2)^{1/2}$ multiplying the right hand side.

It can readily be seen from Eqs. (4) and (5) that, as the mean Mach number is increased, not only is the axial distance over which a given fraction of the distortion attenuation takes place shortened, as is stated in Ref. 1, but the ratio of the circumferential to the irrotational axial velocity perturbation at any location is decreased.

In the analysis presented in Ref. 1, for given values of the normalized slope of the compressor static pressure rise and far upstream velocity perturbation, the axial velocity at the compressor face is fixed. Therefore, for a specified slope and upstream infinity velocity perturbation, the upstream circumferential (and radial in a three-dimensional flow) velocities will be functions of the mean Mach number, as the value of A_n in Eqs. (3–5) depends on the quantity $(1-\bar{M}^2)^{1/2}$. The form of the upstream attenuation is therefore indeed affected by the Mach number.

This result is significant, in that the relative inlet incidence angles and velocities at the first rotor of a compressor without inlet guide vanes do have a first-order dependence on the circumferential velocity component at the compressor face. Thus the distorted flow performance of such a compressor can be strongly affected by the asymmetric inlet circumferential velocity distribution, and it is important to predict this effect correctly.

In connection with this last statement, there is also another aspect of the paper upon which we can comment. It is stated that the local slope of the compressor characteristic under distorted flow was different than the undistorted slope, and that the experimentally measured velocity distributions were not symmetric, as was predicted. The writer feels that these discrepancies may be due in some part to a feature which was not considered in the paper, but which is inherent in compressors without inlet guide vanes. In the analysis, the technique used to represent the response of the compressor to small velocity perturbations was to assume that the local static pressure rise was dependent only on the slope of the compressor performance curve (of static pressure rise versus axial velocity) and the perturbation in axial velocity. In a low Mach number compressor it is indeed a good approximation to assume that the static pressure rise is a function of the axial velocity alone in undistorted flow. In addition, for a compressor which has inlet guide vanes (which tend to suppress the effect of asymmetric inlet circumferential velocity components) it has been found that the distorted performance can also be adequately described by this representation. However, this is not true for a compressor without inlet guide vanes because of the (nonsymmetric) influence of the upstream circumferential velocities induced by the compressor. In this latter case, these velocity components should be accounted for in computing the static pressure rise delivered by the compressor.

Although this has been pointed out in Ref. 3, we can here briefly make the above statements more plausible by again resorting to elementary two-dimensional, quasi-steady arguments concerning the flow in a single stage compressor with no inlet guide vanes. Therefore, in the discussion that follows we will assume that the flow is incompressible and inviscid, and that the compressor blade rows can be replaced by actuator disks which respond quasi-steadily to changes in the inlet flow. Further, we will take the axial gaps between the blade rows to be small enough so that the inter-row cross flows can be neglected (as discussed in Ref. 4, this is usually a very good approximation). It should be emphasized here that the arguments below are presented only to show a simple situation in which the importance of the circumferential components can be clearly seen. Thus, because of the idealized conditions assumed in the arguments, the specific conclusions which result may not be directly applicable to the strongly three-dimensional, nonquasi-steady flow investigated in Ref. 1.

If the methods derived in Ref. 1 were applied to the de-

Received January 28, 1972.
Index category: Airplane and Component Aerodynamics.
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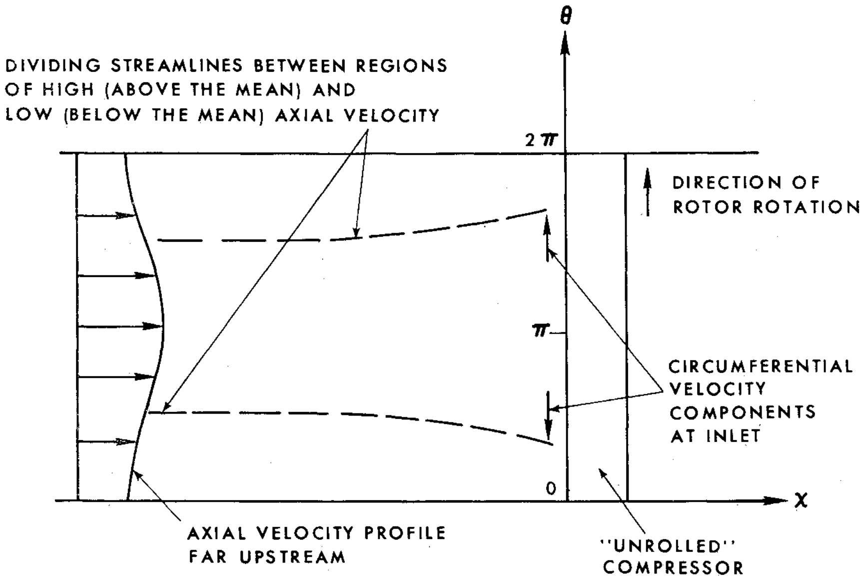

Fig. 1 Sketch of asymmetric circumferential velocity components at compressor inlet.

scription of a two-dimensional flowfield, they would predict that for a far upstream velocity profile of the form

$$U = \bar{U} + \varepsilon_1 \cos\theta,$$

the axial velocity distribution upstream of the compressor would be symmetric about $\theta = \pi$ (using the coordinate notation of Ref. 1), whereas the circumferential velocities would be odd functions about this position. If ε_1 were negative, the perturbation velocities would thus lead to a flow field qualitatively similar to the one sketched in Fig. 1. The compressor rotor is defined in this figure to move in the positive θ-direction. Let us examine the flow at the compressor face at the circumferential locations $\theta = \pi/2$ and $\theta = 3\pi/2$. The linearized analysis presented in Ref. 1 would predict that the axial velocity perturbations are zero at these positions, and the perturbations in static pressure rise across the compressor are thus also zero there. However, the circumferential velocity components predicted by the analysis have maximum amplitudes at these positions. Since they affect both rotor relative incidence angle and relative inlet velocity head to first order, these velocity components can cause first-order changes in the pressure rise delivered by the compressor rotor.

By using Bernoulli's equation in the relative (rotor) frame of reference, as well as the condition that the axial velocity is continuous across a blade row, it can be shown that the perturbation in rotor static pressure rise at any circumferential location is given by

$$\Delta p = \rho\,\omega r_m[v_{re} - v_{0-}] - \rho V_{re} v_{re} \tag{6}$$

In this expression ωr_m is the rotor speed, v_{re} and v_{0-} are the pertubations in circumferential velocity component at rotor exit and rotor inlet, respectively, and V_{re} is the mean circumferential velocity component at the rotor exit. Let us make the assumption that the rotor relative leaving angle is constant (from cascade tests this is known to be a good approximation over a wide range of operating conditions for blade rows of moderate and high solidity). Then, at the specified locations, the value of circumferential velocity perturbation at the rotor exit, v_{re}, is proportional to the axial velocity perturbation and is thus zero, as is the perturbation in static pressure rise across the stator. The equation for the perturbation in stage static pressure rise at $\theta = \pi/2$ and $\theta = 3\pi/2$ under conditions of a symmetric inlet axial velocity profile, as is derived in Ref. 1, can therefore be simplified to

$$\Delta p = -\rho\,\omega r_m v_{0-} \tag{7}$$

Equation (7) says that there will be a larger stage static pressure rise at $\theta = \pi/2$ than at $\theta = 3\pi/2$ even though both have the same axial velocity. From Bernoulli's theorem, both of these locations have the same static pressure at inlet (to first order). Since the assumption in Ref. 1 is that the exit static pressure is constant, we are faced with an inconsistency.

The source of this inconsistency lies in the assumption of the dependence of the static pressure rise on the axial velocity alone, which implies that if the far upstream velocity profile is symmetric all other profiles will also be symmetric. If one in fact examines the compressor without inlet guide vanes on a row by row basis, using an actuator disk analysis as in Refs. 2, 3, or 4, one finds that the presence of a far upstream distortion of the form given in Eq. (1) will lead to the inlet static pressure and axial velocity distributions having both $\sin n\theta$ and $\cos n\theta$ terms—i.e. these flow quantities will not be symmetric.

From the preceding arguments, one can infer that if a two-dimensional calculation of the upstream flow field were carried out under the conditions mentioned, i.e. an ideal incompressible fluid, the nature of the asymmetry would be such as to make the actual axial velocity at $3\pi/2$ less than that at $\pi/2$ (and the static pressure at $3\pi/2$ greater than that at $\pi/2$) in order to be consistent with the exit boundary condition of constant static pressure. Such calculations have been carried out using the method detailed in Ref. 3, and they do indeed give this result.

As mentioned, one cannot form such clear-cut conclusions for more general cases involving the unsteady response of the compressor blading and/or the significant radial flows associated with compressors of low hub to tip ratio. Evidence of this can be seen in the data presented in Ref. 1 (where it is likely that both of these effects were present) which shows an axial velocity asymmetry opposite to that predicted by the above simple arguments. Nevertheless, although the writer concurs with the authors of Ref. 1 when they ascribe some of the disparity between experiment and theory to the unsteady rotor response, it seems that another area in which to investigate the source of this disagreement might be found in the basic asymmetry of the guide-vane-less compressor.

References

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